

# Predictions for the strangeness $S = -3$ and $-4$ baryon-baryon interactions in chiral effective field theory

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## Abstract

The leading order strangeness  $S = -3$  and  $-4$  baryon-baryon interactions are analyzed within chiral effective field theory. The chiral potentials consist of contact terms without derivatives and of one-pseudoscalar-meson exchanges. Assuming SU(3) flavor symmetry those contact terms and the couplings of the pseudoscalar mesons to the baryons are related to the corresponding quantities of the  $S = -1$  hyperon-nucleon channels. Specifically, the values of the pertinent five low-energy constants related to the contact terms are already fixed from our preceeding study of the  $\Lambda N$  and  $\Sigma N$  systems and thus genuine predictions for the  $\Xi\Lambda$ ,  $\Xi\Sigma$ , and  $\Xi\Xi$  interactions can be made. Strong attraction is found in some of the  $S = -3$  and  $-4$  channels, suggesting the possible existence of bound states.

*Key words:* Hyperon-hyperon interaction, Hyperon-nucleon interaction, Effective field theory, Chiral Lagrangian  
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## 1. Introduction

The study of baryon-baryon systems involving strangeness has the potential of considerably deepening our understanding of strong interaction physics. Unfortunately, the experimental information on such systems is rather limited. Though there is at least a fair amount of *concrete* data on the  $\Lambda N$  and  $\Sigma N$  systems, the empirical information on the strangeness  $S = -2$  sector is only of qualitative nature. It consists of limits for the  $\Xi^- p \rightarrow \Xi^- p$  and  $\Xi^- p \rightarrow \Lambda\Lambda$  reaction cross sections [1] and in the binding energy of  ${}_{\Lambda\Lambda}^6\text{He}$  [2], where the latter implies that the  $\Lambda\Lambda$  interaction can be only moderately attractive. Virtually nothing is known about the baryon-baryon interaction in the  $S = -3$  and  $-4$  systems. Indeed, even theoretical investigations on those systems are rather scarce [3,4]. In 1999, the Nijmegen group presented an extension of their meson-exchange hyperon-nucleon ( $YN$ ;  $Y = \Lambda, \Sigma$ ) potential [5] to interaction channels with  $S = -2, -3$ , and  $-4$  [3] invoking SU(3) flavor symmetry arguments. In the actual model calculation, SU(3)<sub>f</sub> symmetry is broken by using the physical values of the involved baryon and meson masses and, in addition, some breaking of the baryon-baryon-meson coupling constants is allowed. The interactions predicted in this way for the  $S = -3$

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and  $-4$  sectors turned out to be fairly strong and attractive and even suggests the existence of bound states in the  $\Xi\Sigma$  and  $\Xi\Xi$  channels [3]. Such strongly attractive  $S = -3$  and  $-4$  baryon-baryon interactions have further interesting implications. For example, Filikin and Gal argued that then fairly light hypernuclei with  $S = -3$  should exist [6,7]. In particular,  ${}_{\Xi\Lambda}^6\text{He}$  could then already mark the onset of nuclear stability for  $\Xi$  hyperons. A strong  $\Xi\Xi$  interaction could induce a first order phase transition from neutron matter to hyperon-rich matter [8] so that, besides ordinary white dwarfs and neutron stars, a new class of compact stars, namely hyperon stars, could exist.

The baryon-baryon interaction of Fujiwara and collaborators [4] is derived in the constituent quark model. It contains the color Fermi-Breit interaction and effective-meson exchanges (pseudoscalar, scalar, vector mesons) between quarks, among others. Also here  $\text{SU}(3)_f$  symmetry plays a key role in extending the model from the  $NN$  and  $YN$  interaction (where free parameters are fixed) to the  $S = -3$  and  $-4$  channels. And also here an explicit flavor-symmetry breaking is introduced, namely in the Fermi-Breit interaction. But in this approach it was found that the baryon-baryon interaction becomes step by step less attractive when going from strangeness  $S = 0$  to  $S = -4$ . In particular, no di-baryon bound states are supported, except for the deuteron.

In this context we present here a first study of the baryon-baryon interaction in the  $S = -3$  and  $-4$  sectors in the framework of *chiral effective field theory* (EFT). Outlined by Weinberg in the early 1990s [9,10] the concepts of chiral EFT have been applied very successfully in the last decade to the  $NN$  (and  $NNN$ ) interaction and to the physics of light nuclei, resulting in a high-precision description of the experimental data, see e.g. Refs. [11,12,13] and references therein. Recently, this framework was utilized by us also for investigating the  $YN$  interactions as well as the baryon-baryon interaction in the  $S = -2$  sector [14,15]. In particular, we showed that the leading order (LO) chiral EFT successfully describes the available  $YN$  scattering data [14]. Also the binding energies of the light hypernuclei are predicted well within chiral EFT [16,17].

The extension of our investigations of the baryon-baryon interaction within chiral EFT to the strangeness  $S = -3$  and  $-4$  sectors is straightforward. The LO potential consists of four-baryon contact terms without derivatives and of one-pseudoscalar-meson exchanges [14,17]. Under the assumption of  $\text{SU}(3)_f$  symmetry the interaction for the  $S = -3$  and  $-4$  baryon-baryon sector depends on the same (five) contact terms that enter also in the  $YN$  interaction. Thus, we can take over the values which were fixed in our study of the  $YN$  sector by a fit to the pertinent ( $\Lambda N$ ,  $\Sigma N$ ) data [14]. Then the interaction in the  $S = -3$  and  $-4$  channels is a genuine prediction that follows from the results of Ref. [14] and  $\text{SU}(3)_f$  symmetry.

The paper is structured as follows: In Sect. II we provide a short overview of the chiral EFT approach with emphasis on its extension to the strangeness  $S = -3$  and  $-4$  sectors. In Sect. III we show results for the coupled  $\Xi\Lambda - \Xi\Sigma$  system and for the  $\Xi\Xi$  channel. Specifically, we present integrated cross sections and effective range parameters for the  $S$ -waves and we compare our results to those of the two potential models mentioned above. The paper ends with a short summary.

## 2. The effective strangeness $S = -3$ and $-4$ baryon-baryon potential

We construct the chiral effective potentials for the strangeness  $S = -3$  and  $-4$  sector at LO using the Weinberg power counting similar to the  $YN$  case considered in [14]. The LO potential consists of four-baryon contact terms without derivatives and of one-pseudoscalar-meson exchanges. Details on the derivation of the LO contact terms for the octet baryon-baryon interaction can be found in Ref. [14,17]. Here we only list the final result for the  $\Xi Y$  and  $\Xi\Xi$  partial wave potentials in the singlet and triplet  $S$ -waves, cf. Table 1. Also the  $S = 0$  and  $-1$  potentials are included in Table 1 for completeness. For convenience in the present paper we express the baryon-baryon potentials in terms of the  $\text{SU}(3)_f$  irreducible representations, see e.g. [18] and also [19,20], so that the contact interaction is given by

$$V^{B_1 B_2 \rightarrow B'_1 B'_2} = \frac{1}{4}(1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) C_{1S0} + \frac{1}{4}(3 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) C_{3S1} . \quad (1)$$

Table 1

Various LO baryon-baryon contact potentials for the  $^1S_0$  and  $^3S_1$  partial waves in the isospin basis. These potentials are flavor symmetric.  $C^{27}$  etc. refers to the corresponding  $SU(3)_f$  irreducible representation [18,20].

	Channel	Isospin	$C_{1S_0}$	Isospin	$C_{3S_1}$
$S = 0$	$NN \rightarrow NN$	1	$C^{27}$	0	$C^{10^*}$
$S = -1$	$\Lambda N \rightarrow \Lambda N$	$\frac{1}{2}$	$\frac{1}{10} (9C^{27} + C^{8_s})$	$\frac{1}{2}$	$\frac{1}{2} (C^{8_a} + C^{10^*})$
	$\Lambda N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{3}{10} (-C^{27} + C^{8_s})$	$\frac{1}{2}$	$\frac{1}{2} (-C^{8_a} + C^{10^*})$
	$\Sigma N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{1}{10} (C^{27} + 9C^{8_s})$	$\frac{1}{2}$	$\frac{1}{2} (C^{8_a} + C^{10^*})$
	$\Sigma N \rightarrow \Sigma N$	$\frac{3}{2}$	$C^{27}$	$\frac{3}{2}$	$C^{10}$
$S = -3$	$\Xi \Lambda \rightarrow \Xi \Lambda$	$\frac{1}{2}$	$\frac{1}{10} (9C^{27} + C^{8_s})$	$\frac{1}{2}$	$\frac{1}{2} (C^{8_a} + C^{10})$
	$\Xi \Lambda \rightarrow \Xi \Sigma$	$\frac{1}{2}$	$\frac{3}{10} (-C^{27} + C^{8_s})$	$\frac{1}{2}$	$\frac{1}{2} (-C^{8_a} + C^{10})$
	$\Xi \Sigma \rightarrow \Xi \Sigma$	$\frac{1}{2}$	$\frac{1}{10} (C^{27} + 9C^{8_s})$	$\frac{1}{2}$	$\frac{1}{2} (C^{8_a} + C^{10})$
	$\Xi \Sigma \rightarrow \Xi \Sigma$	$\frac{3}{2}$	$C^{27}$	$\frac{3}{2}$	$C^{10^*}$
$S = -4$	$\Xi \Xi \rightarrow \Xi \Xi$	1	$C^{27}$	0	$C^{10}$

Table 2

The  $S$ -wave contact terms corresponding to the irreducible representations of  $SU(3)_f$  for various cut-offs. The values of the LECs are in  $10^4 \text{ GeV}^{-2}$ ; the values of  $\Lambda$  are in MeV.

$\Lambda$	550	600	650	700
$C^{27}$	-.0766	-.0763	-.0757	-.0744
$C^{10^*}$	-.0239	-.0182	-.0097	.0013
$C^{10}$	.2336	.2391	.2392	.2501
$C^{8_s}$	.2241	.2839	.3595	.3650
$C^{8_a}$	-.0206	-.0144	-.0097	-.0057

From the relations in Table 1 we can see that for the  $^1S_0$  channel there is a one-to-one correspondence between the  $S = 0$  and  $S = -4$  sectors, i.e. the  $NN$  and the  $\Xi\Xi$  interactions, and also between the  $S = -1$  and  $S = -3$  sectors, i.e. the  $YN$  and  $\Xi Y$  interactions. Thus, if  $SU(3)_f$  symmetry is fully realized then not only the potentials but even the reaction amplitudes would be the same in the corresponding sectors. Of course, the symmetry is no longer fulfilled once physical values for the baryon masses are used in the evaluation of the scattering observables, even for a  $SU(3)_f$  symmetric interaction potential. Note that there is no such correspondence for the  $^3S_1$  channel because here the role of the 10 and  $10^*$  representations is interchanged when going from  $S = 0$  to  $S = -4$  and from  $S = -1$  to  $S = -3$ , respectively.

The actual values of the  $S$ -wave contact terms corresponding to the irreducible representations of  $SU(3)_f$  are summarized in Table 2 for the various cut-offs (as defined more precisely below). These values were fixed by a fit to the  $YN$  data in our earlier study [14]. They are obtained from the low-energy constants listed in Table 2 of that paper via the relations Eqs. (2.8) - (2.13) given therein as well.

The lowest order  $SU(3)_f$  invariant pseudoscalar-meson-baryon interaction Lagrangian with the appropriate symmetries was discussed in [14]. In the isospin basis it reads

$$\begin{aligned}
\mathcal{L} = & -f_{NN\pi} \bar{N} \gamma^\mu \gamma_5 \tau N \cdot \partial_\mu \pi + i f_{\Sigma\Sigma\pi} \bar{\Sigma} \gamma^\mu \gamma_5 \times \Sigma \cdot \partial_\mu \pi \\
& -f_{\Lambda\Sigma\pi} [\bar{\Lambda} \gamma^\mu \gamma_5 \Sigma + \bar{\Sigma} \gamma^\mu \gamma_5 \Lambda] \cdot \partial_\mu \pi - f_{\Xi\Xi\pi} \bar{\Xi} \gamma^\mu \gamma_5 \tau \Xi \cdot \partial_\mu \pi \\
& -f_{\Lambda NK} [\bar{\Lambda} \gamma^\mu \gamma_5 \Lambda \partial_\mu K + \bar{\Lambda} \gamma^\mu \gamma_5 N \partial_\mu K^\dagger] - f_{\Xi\Lambda K} [\bar{\Xi} \gamma^\mu \gamma_5 \Lambda \partial_\mu K_c + \bar{\Lambda} \gamma^\mu \gamma_5 \Xi \partial_\mu K_c^\dagger] \\
& -f_{\Sigma NK} [\bar{\Sigma} \cdot \gamma^\mu \gamma_5 \partial_\mu K^\dagger \tau N + \bar{N} \gamma^\mu \gamma_5 \tau \partial_\mu K \cdot \Sigma] - f_{\Xi\Sigma K} [\bar{\Xi} \cdot \gamma^\mu \gamma_5 \partial_\mu K_c^\dagger \tau \Xi + \bar{\Xi} \gamma^\mu \gamma_5 \tau \partial_\mu K_c \cdot \Sigma] \\
& -f_{NN\eta_8} \bar{N} \gamma^\mu \gamma_5 N \partial_\mu \eta - f_{\Lambda\Lambda\eta_8} \bar{\Lambda} \gamma^\mu \gamma_5 \Lambda \partial_\mu \eta - f_{\Sigma\Sigma\eta_8} \bar{\Sigma} \cdot \gamma^\mu \gamma_5 \Sigma \partial_\mu \eta - f_{\Xi\Xi\eta_8} \bar{\Xi} \gamma^\mu \gamma_5 \Xi \partial_\mu \eta. \quad (2)
\end{aligned}$$

Table 3

Isospin factors  $\mathcal{I}$  for the various one-pseudoscalar-meson exchanges.

Channel	Isospin	$\pi$	$K$	$\eta$
$\Xi\Xi \rightarrow \Xi\Xi$	0	-3	0	1
	1	1	0	1
$\Xi\Lambda \rightarrow \Xi\Lambda$	$\frac{1}{2}$	0	1	1
$\Xi\Lambda \rightarrow \Xi\Sigma$	$\frac{1}{2}$	$-\sqrt{3}$	$-\sqrt{3}$	0
$\Xi\Sigma \rightarrow \Xi\Sigma$	$\frac{1}{2}$	-2	-1	1
	$\frac{3}{2}$	1	2	1

The interaction Lagrangian in Eq. (2) is invariant under  $SU(3)_f$  transformations if the various coupling constants fulfill specific relations which can be expressed in terms of the coupling constant  $f$  and the  $F/(F+D)$ -ratio  $\alpha$  as [18],

$$\begin{aligned}
f_{NN\pi} &= f, & f_{NN\eta_8} &= \frac{1}{\sqrt{3}}(4\alpha - 1)f, & f_{\Lambda NK} &= -\frac{1}{\sqrt{3}}(1 + 2\alpha)f, \\
f_{\Xi\Xi\pi} &= -(1 - 2\alpha)f, & f_{\Xi\Xi\eta_8} &= -\frac{1}{\sqrt{3}}(1 + 2\alpha)f, & f_{\Xi\Lambda K} &= \frac{1}{\sqrt{3}}(4\alpha - 1)f, \\
f_{\Lambda\Sigma\pi} &= \frac{2}{\sqrt{3}}(1 - \alpha)f, & f_{\Sigma\Sigma\eta_8} &= \frac{2}{\sqrt{3}}(1 - \alpha)f, & f_{\Sigma NK} &= (1 - 2\alpha)f, \\
f_{\Sigma\Sigma\pi} &= 2\alpha f, & f_{\Lambda\Lambda\eta_8} &= -\frac{2}{\sqrt{3}}(1 - \alpha)f, & f_{\Xi\Sigma K} &= -f.
\end{aligned} \tag{3}$$

Here  $f \equiv g_A/2F_\pi$ ,  $g_A$  is the axial-vector strength,  $g_A = 1.26$ , which is measured in neutron  $\beta$ -decay and  $F_\pi$  is the weak pion decay constant,  $F_\pi = 92.4$  MeV. For the  $F/(F+D)$ -ratio we adopt here the  $SU(6)$  value ( $\alpha = 0.4$ ) which was already used in our study of the  $YN$  system [14]. The spin-space part of the one-pseudoscalar-meson-exchange potential resulting from the interaction Lagrangian Eq. (2) is in leading order similar to the static one-pion-exchange potential in Ref. [21],

$$V^{B_1 B_2 \rightarrow B'_1 B'_2} = -f_{B_1 B'_1 P} f_{B_2 B'_2 P} \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k})}{\mathbf{k}^2 + m_P^2}, \tag{4}$$

where  $f_{B_1 B'_1 P}$ ,  $f_{B_2 B'_2 P}$  are the appropriate coupling constants as given in Eq. (3) and  $m_P$  is the actual mass of the exchanged pseudoscalar meson. With regard to the  $\eta$  meson we identified its coupling with the octet value, i.e. the one for  $\eta_8$ . The transferred momentum  $\mathbf{k}$  is defined in terms of the final and initial center-of-mass (c.m.) momenta of the baryons,  $\mathbf{p}_f$  and  $\mathbf{p}_i$ , as  $\mathbf{k} = \mathbf{p}_f - \mathbf{p}_i$ . To find the complete LO one-pseudoscalar-meson-exchange potential one needs to multiply the potential in Eq. (4) with the isospin factors  $\mathcal{I}$  given in Table 3.

The  $SU(3)_f$  symmetry of the one-pseudoscalar-meson exchanges is broken by the masses of the pseudoscalar mesons. This is taken into account explicitly in Eq. (4) by taking the appropriate values for  $m_P$ . In case one would consider identical pseudoscalar-meson masses, the corresponding potential obeys the  $SU(3)_f$  relations as shown in the 4th and 6th column of Table 1, respectively.

Finally, for completeness we briefly comment on the used scattering equation. The calculations are done in momentum space. We solve the coupled-channel (non-relativistic) Lippmann-Schwinger (LS) equation,

$$T_{\rho''\rho'}^{\nu''\nu',J}(p'',p';\sqrt{s}) = V_{\rho''\rho'}^{\nu''\nu',J}(p'',p') + \sum_{\rho,\nu} \int_0^\infty \frac{dp p^2}{(2\pi)^3} V_{\rho''\rho}^{\nu''\nu',J}(p'',p) \frac{2\mu_\nu}{q_\nu^2 - p^2 + i\epsilon} T_{\rho\rho'}^{\nu\nu',J}(p,p';\sqrt{s}). \tag{5}$$

The label  $\nu$  indicates the particle channels and the label  $\rho$  the partial wave.  $\mu_\nu$  is the pertinent reduced mass. The on-shell momentum in the intermediate state,  $q_\nu$ , is defined by  $\sqrt{s} = \sqrt{M_{B_{1,\nu}}^2 + q_\nu^2} + \sqrt{M_{B_{2,\nu}}^2 + q_\nu^2}$ . Relativistic kinematics is used for relating the laboratory energy  $T_{\text{lab}}$  of the baryons to the center-of-mass momentum. Suppressing the particle channel label, the partial wave projected potentials  $V_{\rho''\rho'}^J(p'',p')$  are

given in [14]. The LS equation for the coupled channels  $\Xi\Lambda$  and  $\Xi\Sigma$  is solved in the particle basis, in order to incorporate the correct physical thresholds. The potential in the LS equation is cut off with the regulator function  $f^\Lambda(p', p)$ ,

$$f^\Lambda(p', p) = e^{-(p'^4 + p^4)/\Lambda^4}, \quad (6)$$

in order to remove high-energy components of the baryon and pseudoscalar meson fields. We consider again cut-off values in the range of 550...700 MeV as in Ref. [14]. This range is also similar to the one considered in the  $NN$  case, see, e.g. Refs. [22,23]. The cross sections are calculated using the (LSJ basis) partial wave amplitudes, for details we refer to [24,25].

### 3. Results and discussion

The LO chiral EFT interaction for the  $S = -3$  and  $-4$  baryon-baryon sector depends only on those five contact terms that enter also in the  $YN$  interaction. Thus, we can take over the values which were fixed in our study of the  $YN$  sector [14]. Then the interaction in the  $S = -3$  and  $-4$  channels is a genuine prediction that follows from the results of Ref. [14] and  $SU(3)_f$  symmetry.

Results for the  $\Xi^0\Lambda \rightarrow \Xi^0\Lambda$ ,  $\Xi^0\Sigma^- \rightarrow \Xi^-\Lambda$ ,  $\Xi^0\Sigma^- \rightarrow \Xi^-\Sigma^0$ ,  $\Xi^0\Sigma^- \rightarrow \Xi^0\Sigma^-$ , and  $\Xi^0\Sigma^+ \rightarrow \Xi^0\Sigma^+$  scattering cross sections are shown in Fig. 1. Partial waves with total angular momentum up-to-and-including  $J = 2$  are taken into account. The shaded bands show the cut-off dependence. From that figure one observes that the  $\Xi^0\Lambda \rightarrow \Xi^0\Lambda$  and  $\Xi^0\Sigma^+ \rightarrow \Xi^0\Sigma^+$  cross sections are rather large near threshold. Though the cross section for  $\Xi^0\Sigma^- \rightarrow \Xi^-\Lambda$  rises too, in this case it is only due to the phase space factor  $p_{\Xi^-\Lambda}/p_{\Xi^0\Sigma^-}$ . There is a clear cusp effect visible in the  $\Xi^0\Sigma^-$  cross section at  $p_{lab} \approx 106$  MeV/c, i.e. at the opening of the  $\Xi^-\Sigma^0$  channel. On the other hand, we do not observe any sizeable cusp effects in the  $\Xi^0\Lambda$  cross section around  $p_{lab} = 690$  MeV/c, i.e. at the opening of the  $\Xi\Sigma$  channels. The latter is in line with the results reported by the Nijmegen group for their interactions, where a cusp effect in that channel is absent too. In this context we would like to remind the reader that the cusp seen in the corresponding strangeness  $S = -2$  case, namely in the  $\Lambda N$  cross section at the  $\Sigma N$  threshold, is rather pronounced in our chiral EFT interaction [14] but also in conventional meson-exchange potential models [5,26].

Cross section results for the  $\Xi^0\Xi^0$  and  $\Xi^0\Xi^-$  channels are shown in Fig. 2, again as a function of  $p_{lab}$  and with shaded bands that indicate the cut-off dependence.

Results for scattering cross sections at  $p_{lab} = 150$  MeV/c for the various  $\Xi Y$  and  $\Xi\Xi$  channels are listed in Table 4. We also include predictions by other models [3,4] for channels where pertinent results are available in the literature. Those cross sections were evaluated from the effective range parameters given in the corresponding publications.

The table makes clear again that our predictions for the  $\Xi^0\Lambda$  cross section are indeed sizeable. The results of the Nijmegen meson-exchange potential and of the quark model of Fujiwara et al. are significantly smaller in that channel. But we would like to remark that at least one of the interactions from the Nijmegen group (NSC97b) yields values that are already fairly close to ours (with  $\sigma_{\Xi^0\Lambda} \approx 250$  mb at  $p_{lab} = 150$  MeV/c). In the  $\Xi^0\Sigma^+$  channel (which is a pure isospin  $I = 3/2$  state) the cross sections are also rather large. But here the predictions from the other interactions considered are of comparable magnitude.

It is reassuring to see that the variation of our results with the cut-off mass is not very pronounced. In fact, in general the differences in the cross sections are not more than 20% at  $p_{lab} = 150$  MeV/c and, thus, exhibit uncertainties very similar to those that we have observed in our analysis of the  $\Lambda N$  and  $\Sigma N$  cross sections [14]. In this context let us mention that Stoks and Rijken noticed much more pronounced differences between their six NSC97 models when going to the  $S = -3$  and  $-4$  sectors, as reported in [3]. Their cross sections often differ by a factor 2 or 3, or even more. But one should keep in mind that in the Nijmegen potential an  $SU(3)$  breaking, in addition to the one induced by the mass differences of the pseudoscalar mesons, was introduced, cf. Ref. [5].

Results for the  $\Xi^0\Lambda$ ,  $\Xi^0\Sigma^+$ , and  $\Xi\Xi$  scattering lengths and effective ranges are listed in Table 5. Also here values from the  $S = -3$  and  $-4$  baryon-baryon interaction potentials of Refs. [3,4] are included. This table reveals the reason for the sizeable  $\Xi^0\Lambda$  cross section predicted by the chiral EFT interactions, namely a rather

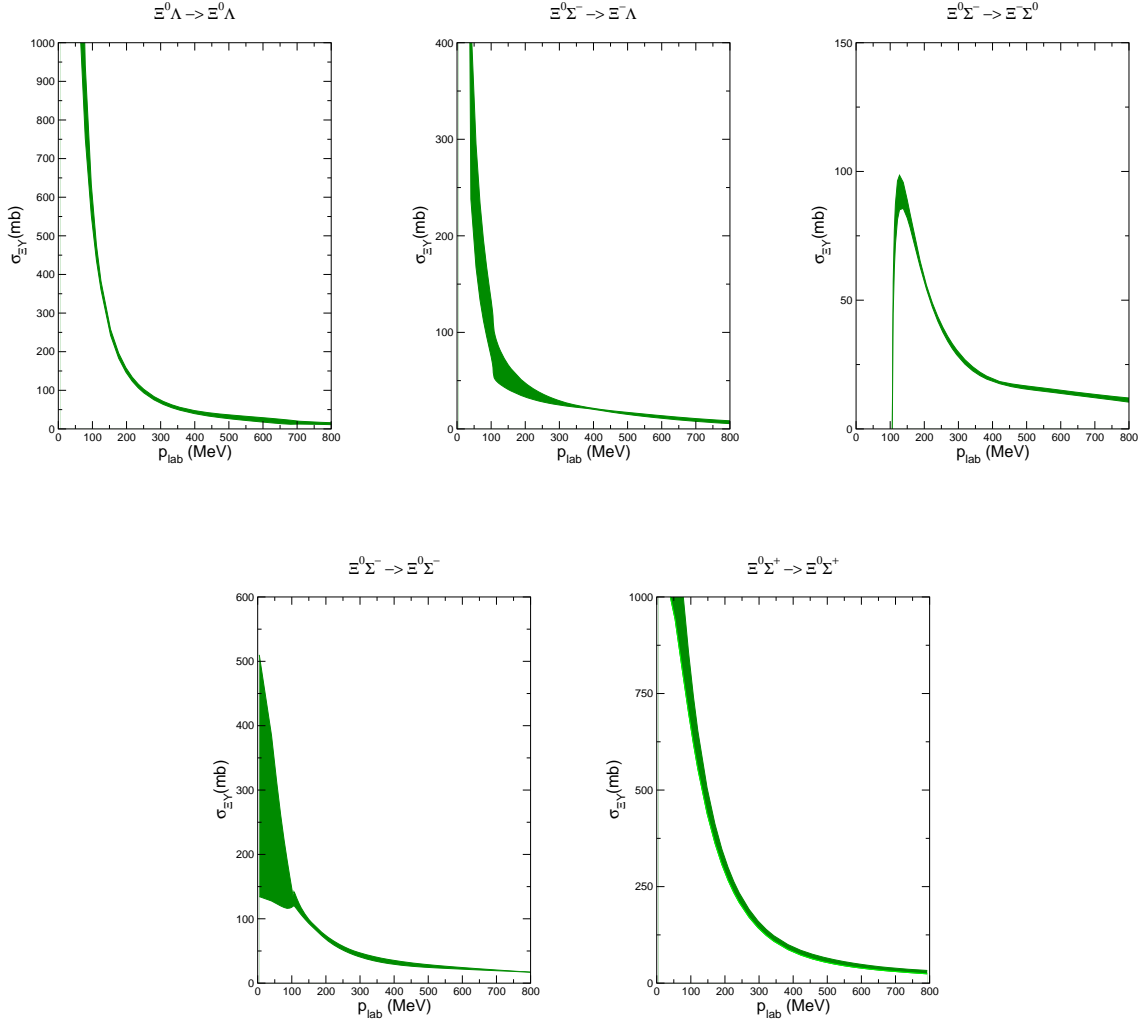


Fig. 1. Total cross sections for various reactions in the strangeness  $S = -3$  sector as a function of  $p_{lab}$ . The shaded band shows the chiral EFT results for variations of the cut-off in the range  $\Lambda = 550 \dots 700$  MeV.

large scattering length in the corresponding  $^1S_0$  partial wave. It is obvious that its value is strongly sensitive to cut-off variations. It even changes sign (in other words, it becomes infinite) within the considered cut-off range. This means that a virtual bound state transforms into a real bound state, where the strongest binding occurs for the cut-off  $\Lambda = 700$  MeV and leads to a binding energy of  $-0.43$  MeV. While this behaviour is interesting per se, one certainly has to stress that in such a case the predictive power of our LO calculation is rather limited. One has to wait for at least an NLO calculation, where we expect that the cut-off dependence will become much weaker so that more reliable conclusions on the possible existence of a virtual or a real bound state should be possible. The  $^1S_0$  scattering lengths of the other potentials suggest also an overall attractive interaction in this partial wave though only a very moderate one.

The results for the  $^3S_1$  state of the  $\Xi^0\Lambda$  channel are fairly similar for all considered interactions. Moreover, with regard to the chiral EFT interaction there is very little cut-off dependence. The  $S$ -waves in the  $\Xi\Sigma$   $I = 3/2$  channel belong to the same ( $10^*$  and  $27$ , respectively) irreducible representations where in the  $NN$  case real ( $^3S_1 - ^3D_1$ ) or virtual ( $^1S_0$ ) bound states exist, cf. Table 1. Therefore, one expects that such states can also occur for  $\Xi\Sigma$ . Indeed, bound states are present for both partial waves in the Nijmegen model, cf.

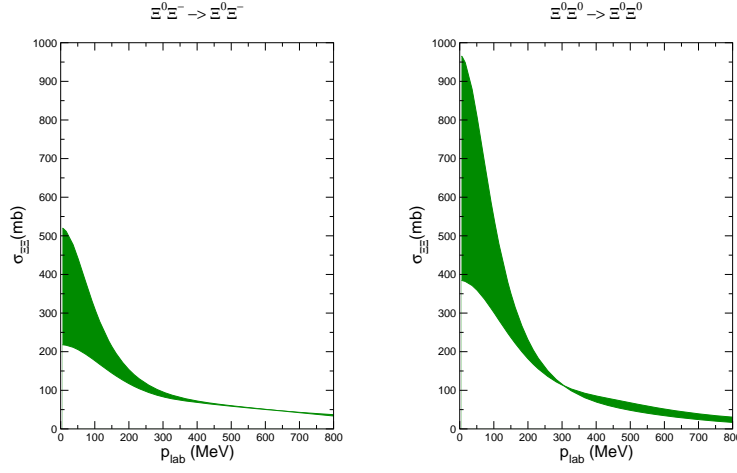


Fig. 2. Total cross sections for the reactions  $\Xi^0 \Xi^- \rightarrow \Xi^0 \Xi^-$  and  $\Xi^0 \Xi^0 \rightarrow \Xi^0 \Xi^0$  as a function of  $p_{lab}$ . The shaded band shows the chiral EFT results for variations of the cut-off in the range  $\Lambda = 550 \dots 700$  MeV.

Table 4

The  $\Xi Y$  and  $\Xi \Xi$  integrated cross sections (in mb) at  $p_{lab} = 150$  MeV/c for various cut-off values  $\Lambda$ . The last columns show results for the Nijmegen potential (NSC97a, NSC97f) [3] and the model by Fujiwara et al. (fss2) [4].

	EFT				NSC97a	NSC97f	fss2
$\Lambda$ (MeV)	550	600	650	700			
$\Xi^0 \Lambda \rightarrow \Xi^0 \Lambda$	254	269	268	262	38.5	66.7	29.0
$\Xi^0 \Sigma^- \rightarrow \Xi^- \Lambda$	66.4	58.8	49.5	40.9			
$\Xi^0 \Sigma^- \rightarrow \Xi^- \Sigma^0$	82.0	83.0	85.2	89.0			
$\Xi^0 \Sigma^- \rightarrow \Xi^0 \Sigma^-$	92.8	99.0	99.0	96.8			
$\Xi^0 \Sigma^+ \rightarrow \Xi^0 \Sigma^+$	430	469	484	492	725	357	400
$\Xi^0 \Xi^0 \rightarrow \Xi^0 \Xi^0$	351	302	261	237	420	250	62.3
$\Xi^0 \Xi^- \rightarrow \Xi^0 \Xi^-$	231	184	160	145	228	151	437

the discussion in Sect. III.B in Ref. [3]. Their presence is reflected in the positive and fairly large singlet and triplet scattering lengths for  $\Xi^0 \Sigma^+$ , cf. Table 5. The chiral EFT interaction has positive scattering lengths of comparable magnitude for  $^1S_0$ , for all cut-off values, and therefore bound states, too. These binding energies lie in the range of  $-2.23$  MeV ( $\Lambda = 550$  MeV) to  $-6.15$  MeV (700 MeV). In the  $^3S_1 - ^3D_1$  partial wave the attraction is obviously not strong enough to form a bound state. The same is the case (but for both  $S$  waves) for the quark model fss2 of Fujiwara et al. [4].

The  $^1S_0$  state of the  $\Xi \Xi$  channel belongs also to the 27plet irreducible representation and also here the Nijmegen as well as the chiral EFT interactions produce bound states (see also [27]). In our case the binding energies lie in the range of  $-2.56$  MeV ( $\Lambda = 550$  MeV) to  $-7.28$  MeV (700 MeV). The predictions of both approaches for the  $^3S_1$  scattering length are comparable. The quark model of Fujiwara et al. exhibits a different behavior for the  $\Xi \Xi$  channel, see the last column in Table 5. The small and negative  $^1S_0$  scattering length signals an interaction that is only moderately attractive. The large and positive scattering length in the  $^3S_1 - ^3D_1$  partial wave, produced by that potential model, is usually a sign for the presence of a bound state, though according to the authors this is not the case for this specific interaction. Nevertheless, the  $\Xi^0 \Xi^-$  cross section predicted by the quark model is significantly larger than the results of our chiral EFT interaction as well as those of the Nijmegen meson-exchange potential, see, Table 4.

Table 5

Selected  $\Xi Y$  and  $\Xi\Xi$  singlet and triplet scattering lengths  $a$  and effective ranges  $r$  (in fm) for various cut-off values  $\Lambda$ . The last columns show results for the Nijmegen potential (NSC97a, NSC97f) [3] and the model by Fujiwara et al. (fss2) [4].

	EFT				NSC97a	NSC97f	fss2
$\Lambda$ (MeV)	550	600	650	700			
$a_s^{\Xi\Lambda}$	-33.5	35.4	12.7	9.07	-0.80	-2.11	-1.08
$r_s^{\Xi\Lambda}$	1.00	0.93	0.87	0.84	4.71	3.21	3.55
$a_t^{\Xi\Lambda}$	0.33	0.33	0.32	0.31	0.54	0.33	0.26
$r_t^{\Xi\Lambda}$	-0.36	-0.30	-0.29	-0.27	-0.47	2.79	2.15
$a_s^{\Xi^0\Sigma^+}$	4.28	3.45	2.97	2.74	4.13	2.32	-4.63
$r_s^{\Xi^0\Sigma^+}$	0.96	0.90	0.84	0.81	1.46	1.17	2.39
$a_t^{\Xi^0\Sigma^+}$	-2.45	-3.11	-3.57	-3.89	3.21	1.71	-3.48
$r_t^{\Xi^0\Sigma^+}$	1.84	1.72	1.70	1.70	1.28	0.96	2.52
$a_s^{\Xi\Xi}$	3.92	3.16	2.71	2.47	17.28	2.38	-1.43
$r_s^{\Xi\Xi}$	0.92	0.85	0.79	0.75	1.85	1.29	3.20
$a_t^{\Xi\Xi}$	0.63	0.59	0.55	0.52	0.40	0.48	3.20
$r_t^{\Xi\Xi}$	1.04	1.05	1.08	1.11	3.45	2.80	0.22

#### 4. Summary

In this letter we have presented leading-order results for the  $\Xi Y$  ( $Y = \Lambda, \Sigma$ ) and  $\Xi\Xi$  interactions obtained within a chiral effective field theory approach based on Weinberg's power counting, derived analogous to the  $YN$  potential presented in [14], by relating the strangeness  $S = -3$  and  $-4$  baryon-baryon interactions via  $SU(3)$  flavor symmetry to the one in the  $YN$  system.

The LO chiral potential consists of two pieces: firstly, the longer-ranged one-pseudoscalar-meson exchanges and secondly, shorter-ranged four-baryon contact terms without derivatives. All the occurring five contact terms were already fixed in our study of the  $YN$  interaction so that genuine predictions can be made for the  $S = -3$  and  $-4$  baryon-baryon interactions based on chiral EFT and the assumed  $SU(3)_f$  symmetry. The reaction amplitude is obtained by solving a regularized coupled-channel Lippmann-Schwinger equation for the LO chiral potential. We used an exponential regulator function to regularize the potential and applied cut-offs in the range between 550 and 700 MeV.

It will be interesting to see whether the new facilities J-PARC (Tokai, Japan) and FAIR (Darmstadt, Germany) allow access to empirical information about the interaction in the  $S = -3$  and  $-4$  sector. Such information could come from formation experiments of corresponding hypernuclei or from proton-proton and antiproton-proton collisions at such high energies that pairs of baryons with strangeness  $S = -3$  or  $S = -4$  can be produced. There is also a possibility to find signals for strange di-baryon states in heavy-ion collisions [28] which likewise would provide information on the corresponding baryon-baryon interaction. The chiral EFT developed here could then be used to analyze these upcoming data in a model-independent way. It would be also interesting to see how these interactions can be tested in multi-strange three-baryon systems recently explored in lattice QCD [29].



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